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Splitting Triplet and Doublet in Extra Dimensions

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Abstract

A novel mechanism to realize the triplet-doublet splitting in supersymmetric SU(5) grand unified theories is proposed in the framework of higher dimensional theories where chiral multiplets are localized due to kink configuration of a SU(5) singlet. An adjoint Higgs field which spontaneously breaks the SU(5) gauge symmetry is assumed to be involved with the localization process, splitting the wave functions of the color-triplet Higgs from its doublet counterpart. The resulting effective four-dimensional theory does not possess manifest SU(5) invariance. By adjusting couplings, the doublet mass can be exponentially suppressed. We also show that dimension 5 proton decay from triplet Higgs exchange can be suppressed to a negligible level.

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Supersymmetric (SUSY) grand unified theory (GUT) [1] is a very interesting framework for unified description of particles and gauge forces. The gauge hierarchy problem inherent to GUT is solved by introduction of SUSY. The gauge interaction in the standard model follows a unified description at very high energy scale. It is very suggestive that experimental data support this view [2].

A simple class of SUSY GUTs, however, suffers from two severe problems. The first one is the triplet-doublet splitting problem (in $SU(5)$ terminology). A Higgs doublet in the standard model is associated with its color-triplet counterpart to make a complete 5 (or 5^*) representation in $SU(5)$. (A similar argument applies for a larger GUT group.) The Higgs doublet mass should be around the weak scale, while the triplet should be superheavy. This splitting is quite non-trivial. In the minimal SUSY $SU(5)$, it is achieved by fine tuning parameters in the superpotential of order 10^{14}. This is possible, but obviously unsatisfactory. The second problem is about proton decay. In SUSY GUTs, exchange of the color-triplet Higgs superfields generically induces dimension-five proton-decay operators [3], which will predict too short life time of the proton, inconsistent with experimental results [4].

One of the solutions for the first problem is the so-called missing doublet mechanism based on group theoretical grounds. Examples are given in $SU(5)$ GUT [5] and in a product GUT such as $SU(5) \times U(3)_H$ [6]. The second problem of the proton decay is solved in the product GUT and also in an axionic extension of the missing doublet mechanism [7].

In this paper, we would like to explore an alternative approach to the above two problems, which is inherently higher dimensional. The point is that the effective four-dimensional theory obtained after dimensional reduction need not respect $SU(5)$ symmetry if $SU(5)$ gauge symmetry is *non-trivially* broken in the extra dimension(s). To realize this point, we utilize the localization mechanism of chiral multiplets in five dimensions. Here we shall consider the situation that, in addition to a GUT singlet which has a non-trivial field configuration along the compactified 5th dimension, an adjoint Higgs field responsible for the GUT gauge symmetry breaking couples to the Higgs multiplets and its conjugate fields in five dimensions. Then the wave function of the triplet and that of the doublet are localized at different places in the extra dimension and are not related with each other by the $SU(5)$ transformation. We will show that the superpotential of the resulting effective four-dimensional theory does not possess manifest $SU(5)$ invariance, while the gauge coupling unification is unchanged. Note that recently there have been very interesting proposals by [8–10]. There the parity assignment in five dimensions does not commute with the $SU(5)$ symmetry and thus the resulting four-dimensional theory does not have apparent $SU(5)$ invariance.

Using the mechanism outlined above, we shall try to explain the triplet-doublet splitting of the Higgs multiplet. We will also show that the dimension five operators inducing the proton decay can become negligibly small.

We first review the localization of chiral multiplets in five dimensional theory. The localization of fermion wave functions under solitonic backgrounds is known for long [11,12]. The mechanism was used to explain the hierarchical mass structure of quarks and leptons and the suppression of proton decay [13–15] in the context of large extra dimensions [16]. See also [17,18] for fermion localization in the Randall-Sundrum scenario [19]. Supersymmetric extension was discussed in [20].

We follow the formalism of [21] and [20] in which four-dimensional $N = 1$ supersymmetry is manifest. Throughout this paper, we consider the five dimensional case where we have only one extra dimension. We use a unit that the fundamental five-dimensional Planck scale $M_* = 1$. Let us denote the coordinate of the fifth dimension by y and introduce five-dimensional fields as one-parameter families of four-dimensional chiral superfields $\Phi(y)$ and its charge conjugated counterpart $\Phi^C(y)$. In components they are written as $\Phi(y) = \phi(y) + \theta\psi(y) + \theta^2 F(y)$ and $\Phi^C(y) = \phi^C(y) + \theta\psi^C(y) + \theta^2 F^C(y)$, with θ being the Grassmann odd coordinate in superspace. Consider the Lagrangian

$$L = \int dy \left\{ \int d^4\theta \left(\Phi(y)^\dagger \Phi(y) + \Phi^C(y)^\dagger \Phi^C(y) \right) + \int d^2\theta \Phi^C(y) \left[\partial_y + M(y) \right] \Phi(y) + H.c. \right\}, \quad (1)$$

where $M(y)$ is a field dependent *mass term*

$$M(y) = \Xi(y) + M. \quad (2)$$

Here Ξ is another chiral superfield and M is a mass parameter. In the following we assume that only the scalar component of Ξ has some non-vanishing configuration and drop higher components. Note that the Lagrangian has only $N = 1$ supersymmetry in four dimensions, while $N = 1$ supersymmetry in five dimensions, which corresponds to $N = 2$ in four dimensions, is assumed to be broken, allowing the coupling of Ξ with Φ and Φ^C. This assumption is crucial in our purpose.

We make Kaluza-Klein decomposition. Equations for zero modes (massless modes in four-dimensional sense) are for $\Phi(y)$

$$\left(\partial_y + M(y)\right) \phi(y) = 0, \quad \left(\partial_y + M(y)\right) \psi(y) = 0, \quad (3)$$

and for $\Phi^C(y)$

$$\left(\partial_y - M(y)\right) \phi^C(y) = 0, \quad \left(\partial_y - M(y)\right) \psi^C(y) = 0. \quad (4)$$

Now suppose that the field Ξ has a kink configuration along the 5th dimension. We approximate it as

$$\Xi(y) = 2\mu^2 y \quad (5)$$

with $\mu^2 > 0$. And we assume that only the zero modes for ϕ and ψ survive and those for ϕ^C and ψ^C do not. This may be achieved by appropriate boundary conditions. Then the wave functions are localized around $l \equiv -M/2\mu^2$ with the following form:

$$\phi(y) = \psi(y) = \left(\frac{2\mu^2}{\pi}\right)^{1/4} \exp\left[-\mu^2 (y - l)^2\right]. \quad (6)$$

Let us next explain our model and notations. It is natural to assume that the five-dimensional Planck scale M_* is in between the GUT scale M_{GUT} and the four-dimensional Planck scale M_{Pl}. And the compactified 5th dimension is assumed to have length about one order of magnitude (or so) larger than the inverse of the fundamental scale M_*. To simplify the following argument, we will not distinguish the two orders of magnitude difference between M_{GUT} and M_{Pl}, and take a unit $M_{GUT} \approx M_{PL} \approx M_* = 1$.

In addition to the Higgs multiplets $H(5)$ and $\bar{H}(5^*)$, we introduce their conjugate multiplets $H^C(5^*)$ and $\bar{H}^C(5)$ respectively. The relevant part of the superpotential is

$$W = \int d^2\theta \left\{ H^C(y) \left[\partial_y + f\Xi(y) + g\Sigma(y) + M \right] H(y) \right.$$
$$\left. + \bar{H}^C(y) \left[\partial_y + \bar{f}\Xi(y) + \bar{g}\Sigma(y) + \bar{M} \right] \bar{H}(y) \right\}, \tag{7}$$

where Ξ is a $SU(5)$ singlet, Σ is an adjoint Higgs field responsible for spontaneous breakdown of the $SU(5)$ gauge symmetry, f, g, $\bar{f}$, $\bar{g}$ are Yukawa couplings and M, $\bar{M}$ are mass parameters. We assume that Ξ has a non-trivial classical configuration along the fifth dimension. To be specific, we suppose in 5×5 matrix notation

$$\Xi(y) = 2\xi^2 (y - y_0) \begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix}. \tag{8}$$

The effect of y_0 in $\Xi(y)$ configuration is absorbed by redefinition of M and $\bar{M}$ and thus we can take $y_0 = 0$. As for $\Sigma(y)$, we assume

$$\Sigma(y) = 2\sigma^2 y \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix} + \Sigma \begin{pmatrix} 2 & & & & \\ & 2 & & & \\ & & 2 & & \\ & & & -3 & \\ & & & & -3 \end{pmatrix}. \tag{9}$$

Note that the non-trivial dependence on y in the first term of the above equation is not essential in the subsequent argument. In fact, the splitting of the triplet and the doublet takes place even for a flat configuration of $\Sigma(y)$ along the extra dimension, as we will see shortly.

It follows from Eqs. (8) and (9) that

$$f\Xi(y) + g\Sigma(y) + M = \begin{pmatrix} 2\mu_T^2 y + M_T & & & & \\ & 2\mu_T^2 y + M_T & & & \\ & & 2\mu_T^2 y + M_T & & \\ & & & 2\mu_u^2 y + M_u & \\ & & & & 2\mu_u^2 y + M_u \end{pmatrix}, \tag{10}$$

where

$$\mu_T^2 \equiv f\xi^2 + 2g\sigma^2, \tag{11}$$
$$\mu_u^2 \equiv f\xi^2 - 3g\sigma^2, \tag{12}$$
$$M_T \equiv M + 2g\Sigma, \tag{13}$$
$$M_u \equiv M - 3g\Sigma. \tag{14}$$

A similar expression is obtained for $\bar{H}$ with

$$\mu_{\bar{T}}^2 \equiv \bar{f}\xi^2 + 2\bar{g}\sigma^2, \tag{15}$$
$$\mu_d^2 \equiv \bar{f}\xi^2 - 3\bar{g}\sigma^2, \tag{16}$$
$$M_{\bar{T}} \equiv \bar{M} + 2\bar{g}\Sigma, \tag{17}$$
$$M_d \equiv \bar{M} - 3\bar{g}\Sigma. \tag{18}$$

After the $SU(5)$ breakdown, the 5 dimensional Higgs H is decomposed into the color triplet H_T and the doublet Higgs H_u. In our setting the zero mode wave functions for H_T and H_u become

$$\psi_T(y) = \left(\frac{2\mu_T^2}{\pi}\right)^{1/4} \exp\left[-\mu_T^2\left(y - l_T\right)^2\right], \tag{19}$$
$$\psi_u(y) = \left(\frac{2\mu_u^2}{\pi}\right)^{1/4} \exp\left[-\mu_u^2\left(y - l_u\right)^2\right], \tag{20}$$

where $l_T = -M_T/2\mu_T^2$ and $l_u = -M_u/2\mu_u^2$. Similarly we obtain the wave functions for $H_{\bar{T}}$ and H_d, by replacing μ_T and μ_u with $\mu_{\bar{T}}$ and μ_d. Explicitly they are

$$\psi_{\bar{T}}(y) = \left(\frac{2\mu_{\bar{T}}^2}{\pi}\right)^{1/4} \exp\left[-\mu_{\bar{T}}^2\left(y - l_{\bar{T}}\right)^2\right], \tag{21}$$
$$\psi_d(y) = \left(\frac{2\mu_d^2}{\pi}\right)^{1/4} \exp\left[-\mu_d^2\left(y - l_d\right)^2\right] \tag{22}$$

with $l_{\bar{T}} = -M_{\bar{T}}/2\mu_{\bar{T}}^2$ and $l_d = -M_d/2\mu_d^2$.

Let us next consider the triplet-doublet splitting in this setting. What we would like to realize is a situation that the triplet is superheavy while the doublet is light. To achieve this, we consider the following two cases.

The first case is that H and $\bar{H}$ have a five dimensional mass term and overlaps of the wave functions themselves determine the size of the masses in four dimensions. The five dimensional mass term is given by the following superpotential

$$\int d^2\theta \int dy M' H(y)\bar{H}(y), \tag{23}$$

where M' is the mass parameter of the order of M_*. In this case the smallness of the doublet mass should be explained by the small overlap of the two wave functions $\psi_u(y)$ and $\psi_d(y)$. The pattern of the wave functions we wish to get is illustrated in Fig. 1. Requiring the resulting mass term does not exceed the electroweak scale, we find

$$\int dy \psi_u(y)\psi_d(y) \lesssim 10^{-(14\sim16)}. \tag{24}$$

The right hand side of the above equation is computed as

$$\int dy \psi_u(y)\psi_d(y) = \left(\frac{4\mu_u^2\mu_d^2}{(\mu_u^2+\mu_d^2)^2}\right)^{1/4} \exp\left[-\frac{\mu_u^2\mu_d^2}{(\mu_u^2+\mu_d^2)}(l_u - l_d)^2\right], \tag{25}$$

from which we obtain the constraint

$$|l_u - l_d| \gtrsim 6 \left(\frac{(\mu_u^2 + \mu_d^2)}{\mu_u^2 \mu_d^2} \right)^{1/2} . \tag{26}$$

It is possible to arrange the locations of the wave functions to satisfy the above constraint. It only requires a tuning of the parameters of order at most 10. A potential problem of this configuration where the triplets are located in between the two doublets is that the triplet Higgs substantially couples to the quarks and leptons when they are located to yield realistic Yukawa couplings and thus a mechanism to suppress the dimension five proton decay given below will not work.

The second possibility we want to discuss here is to introduce a new singlet field $S(y)$. Suppose that, instead of the direct mass term considered above, the singlet has the following Yukawa interaction with H and $\bar{H}$

$$\lambda \int d^2\theta \int dy S(y) H(y) \bar{H}(y). \tag{27}$$

The presence of Eq. (27) and the absence of Eq. (23) may be a consequence of some symmetry. Assume that the S field is localized nearby H_T and $H_{\bar{T}}$ peaks while the peaks of H_u and H_d are located far away. The configuration is schematically depicted in Fig. 2. The realization requires again some tuning of the parameters of order at most 10. This should be compared to the huge fine tuning of 10^{14} or so which is needed in the conventional approach to the minimal SUSY $SU(5)$. Separation of S and H_u, H_d will imply small Yukawa coupling for them in four dimensions. Thus the dimensional reduction yields the following effective theory

$$\int d^2\theta \left[\lambda_3 S H_T H_{\bar{T}} + \lambda_2 S H_u H_d \right], \tag{28}$$

where λ_2 and λ_3 are Yukawa couplings which we assume to achieve $\lambda_2 \lesssim 10^{-14}$ and $\lambda_3 \sim O(1)$. Note that in the resulting four-dimensional theory the $SU(5)$ symmetry is not respected. Recall that This is due to the fact that the the $SU(5)$ breaking Higgs field nontrivial couplings to the Higgs multiplets in five dimensions, which makes the wave functions of the triplet and the doublet completely different.

On the other hand, the (unbroken) gauge fields in the standard model have flat configuration in the extra dimension and hence the gauge coupling unification is guaranteed at tree level. It is interesting to see how Kaluza-Klein modes affect the coupling unification, which is beyond the scope of this paper.

If S condensates around or just below the GUT scale, then H_T, $H_{\bar{T}}$ acquires a GUT scale mass, while the doublet remains very light. Thus the triplet-doublet splitting is achieved. The singlet S will become superheavy and thus it does not destabilize the gauge hierarchy at quantum level [22]. Note that the splitting in our setting is not automatic, but requires some adjustment of the parameters to fix the locations of the wave functions. In this sense, our mechanism is similar to the conventional case of the minimal SUSY $SU(5)$. However, the amount of the adjustment required here is not very huge, thanks to the Gaussian behaviour of the wave functions.

Realistic Yukawa couplings in the quark sector can be obtained by using the localization of the chiral multiplets. Hierarchically small Yukawa couplings are explained by small overlaps of the wave functions. An example is given in [20] in SUSY $SU(5)$. Another example in which a variant of Fritzsch-type texture [23] is obtained will be discussed elsewhere [24].

Another interesting point we should stress is about proton decay. In SUSY GUTs the dimension five operators which cause proton decay can be induced by exchange of the triplet Higgs multiplets. As was discussed earlier, this is one of the major obstacles to build a realistic SUSY GUT model. In most cases, the Yukawa couplings of the triplet Higgs are similar in size to those of the doublet partner. In our setting, however, this may not be the case because the $SU(5)$ invariance is not manifest in the low-energy four-dimensional theory. It provides a very intriguing resolution of the proton decay problem. In fact, the Yukawa couplings between the triplet Higgs and quarks/leptons can be suppressed if the distance between the locations of quarks/leptons and the location of the triplet Higgs is so large that the convolution of the wave functions becomes sufficiently small. Then the proton decay by the triplet Higgs exchange can be suppressed to a negligible level.

To summarize, we have proposed a new approach to the triplet-doublet splitting in SUSY GUT based on higher dimensional theories. If the GUT symmetry breaking is inherently higher dimensional, the four-dimensional effective theory obtained from dimensional reduction need not have GUT symmetry phase. Then the mass of the doublet Higgs can be completely different from that of the triplet counterpart. We have used the mechanism of field localization in the extra dimension under kink configuration to realize this idea in field theoretic approach. The GUT braking Higgs is assumed to have non-vanishing couplings to the 5 and 5^* Higgs multiplets and thus plays a non-trivial role in the localization process. This splits the wave functions of the doublet Higgses from those of the triplet. To obtain very light doublet and superheavy triplet, we have considered the two cases. The latter case with introduction of a new singlet is particularly interesting, since it can provide realistic quark masses by appropriate localization of quark wave functions. Furthermore we can explain the suppression of the proton decay through colored-triplet exchange by making the Yukawa couplings of the triplet to quarks and leptons very small. This can be achieved if the triplet Higgs fields are localized far away from the positions of the quarks and leptons. This is another advantage of our mechanism that the four-dimensional theory does not have the GUT symmetric phase.

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REFERENCES

[1] S. Dimopoulos and H. Georgi, Nucl. Phys. B **193** (1981) 150; N. Sakai, Z. Phys. C **11** (1981) 153.

[2] C. Giunti, C. W. Kim and U. W. Lee, Mod. Phys. Lett. A **6** (1991) 1745. U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B **260** (1991) 447. P. Langacker and M. Luo, Phys. Rev. D **44** (1991) 817.

[3] N. Sakai and T. Yanagida, Nucl. Phys. B **197** (1982) 533. S. Weinberg, Phys. Rev. D **26** (1982) 287.

[4] T. Goto and T. Nihei, Phys. Rev. D **59** (1999) 115009 [hep-ph/9808255].

[5] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, Phys. Lett. B **115** (1982) 380.

[6] T. Hotta, K. I. Izawa and T. Yanagida, Phys. Rev. D **53** (1996) 3913 [hep-ph/9509201]; Phys. Rev. D **54** (1996) 6970 [hep-ph/9602439].

[7] J. Hisano, T. Moroi, K. Tobe and T. Yanagida, Phys. Lett. B **342** (1995) 138 [hep-ph/9406417].

[8] Y. Kawamura, Prog. Theor. Phys. **103** (2000) 613 [hep-ph/9902423]; hep-ph/0012352.

[9] G. Altarelli and F. Feruglio, hep-ph/0102301.

[10] L. Hall and Y. Nomura, hep-ph/0103125.

[11] R. Jackiw and C. Rebbi, Phys. Rev. D **13** (1976) 3398.

[12] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B **125** (1983) 136.

[13] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61** (2000) 033005 [hep-ph/9903417].

[14] E. A. Mirabelli and M. Schmaltz, Phys. Rev. D **61** (2000) 113011 [hep-ph/9912265].

[15] G. Dvali and M. Shifman, Phys. Lett. B **475** (2000) 295 [hep-ph/0001072].

[16] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B **429** (1998) 263 [hep-ph/9803315]; Phys. Rev. D **59** (1999) 086004 [hep-ph/9807344].

[17] Y. Grossman and M. Neubert, Phys. Lett. B **474** (2000) 361 [hep-ph/9912408].

[18] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. D **62** (2000) 084025 [hep-ph/9912498].

[19] L. Randall and R. Sundrum, Phys. Rev. Lett. **83** (1999) 3370 [hep-ph/9905221].

[20] D. E. Kaplan and T. M. Tait, JHEP**0006** (2000) 020 [hep-ph/0004200].

[21] N. Arkani-Hamed, L. Hall, D. Smith and N. Weiner, Phys. Rev. D **63** (2001) 056003 [hep-ph/9911421].

[22] J. Polchinski and L. Susskind, Phys. Rev. D **26** (1982) 3661; H. P. Nilles, M. Srednicki and D. Wyler, Phys. Lett. B **124** (1983) 337.

[23] H. Fritzsch, Phys. Lett. B **73** (1978) 317; Nucl. Phys. B **155** (1979) 189.

[24] M. Kakizaki and M. Yamaguchi, in preparation.

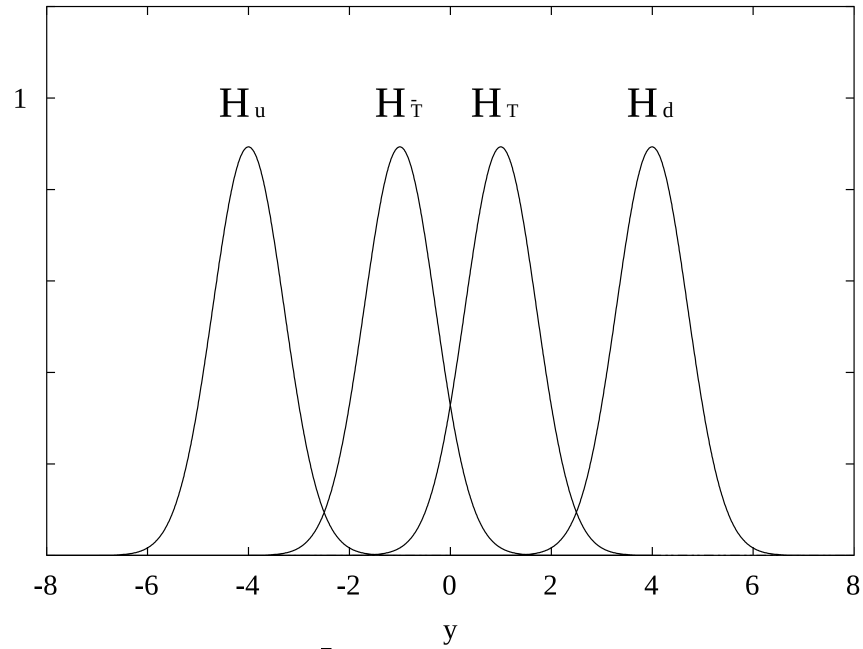

FIG. 1. Higgs profiles for $f = \bar{f} = g = -\bar{g} = 1, \sigma = 0, \xi = M' = M_*$ and, $M = -\bar{M} = -\Sigma = 2M_*$, so that triplet Higgs masses $\sim 0.14 M_*$ and doublet ones $\sim 1.3 \times 10^{-14} M_*$.

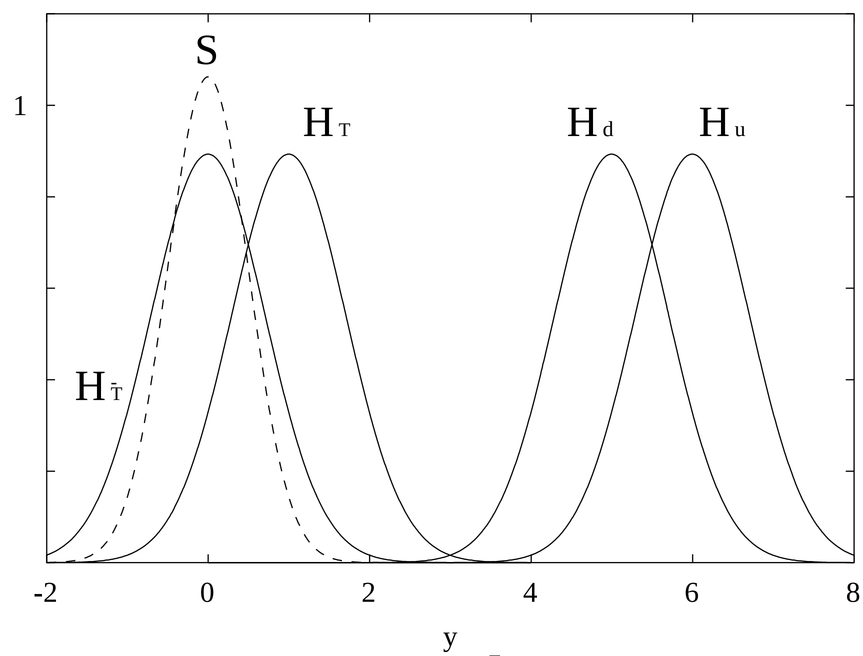

FIG. 2. Higgs and singlet field profiles for $f = \bar{f} = g = \bar{g} = 1, \sigma = 0, \xi = M_*, \Sigma = 2M_*$, $M = -6M_*, \bar{M} = -4M_*, \mu_s = \sqrt{2} M_*$ and, $l_s = 0$, so that $\lambda_2 \sim 3.3 \times 10^{-14}$ and $\lambda_3 \sim 0.35$. A singlet S which is localized around the locations of the triplets is introduced.